Identifying and Estimating Causal Effects with Incomplete Causal Information

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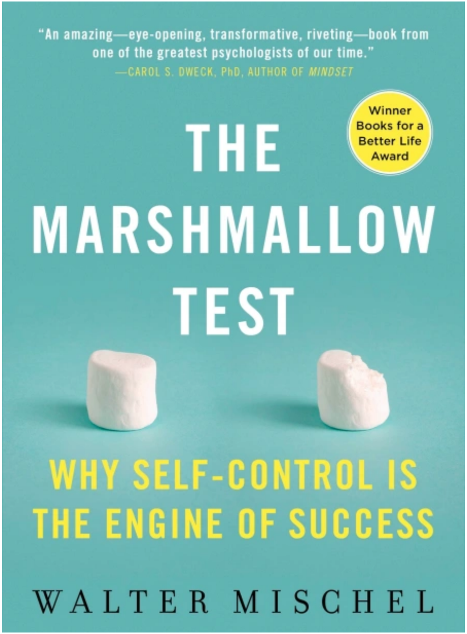
University of Washington

some joint work with F. Richard Guo,
Andrea Rotnitzky, Marloes Maathuis, Leonard Henckel

Stanford Marshmallow Experiment



Marshmallow Test

?

Marshmallow Test

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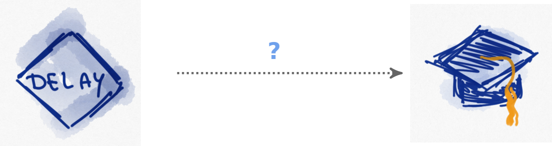

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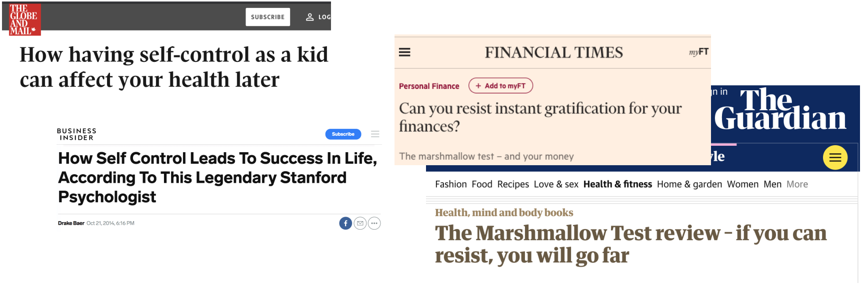
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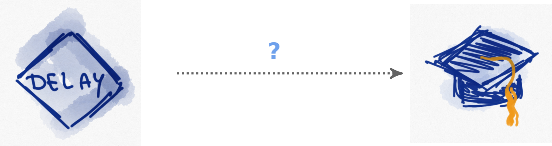


Should we train the delay of gratification?

Marshmallow Test

Assumptions:

- Access to observational data + domain knowledge.
- Do we know all variables that explain or moderate link?
- Do we know all relationships between these variables?

Causal Relationships

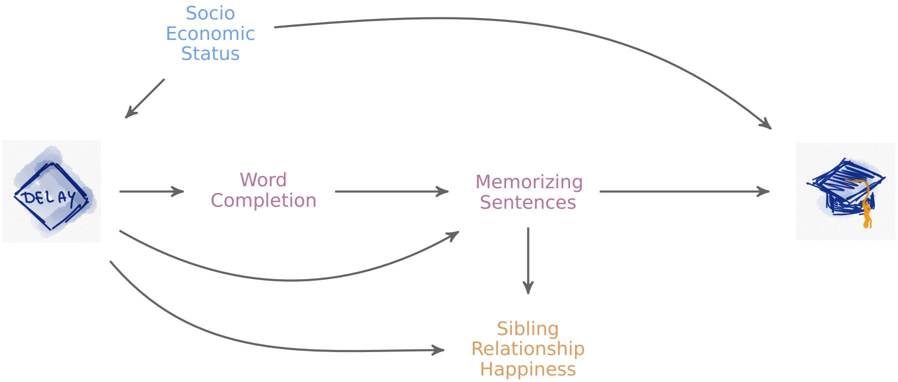
Socio
Economic
Status

Word
Completion

Memorizing
Sentences

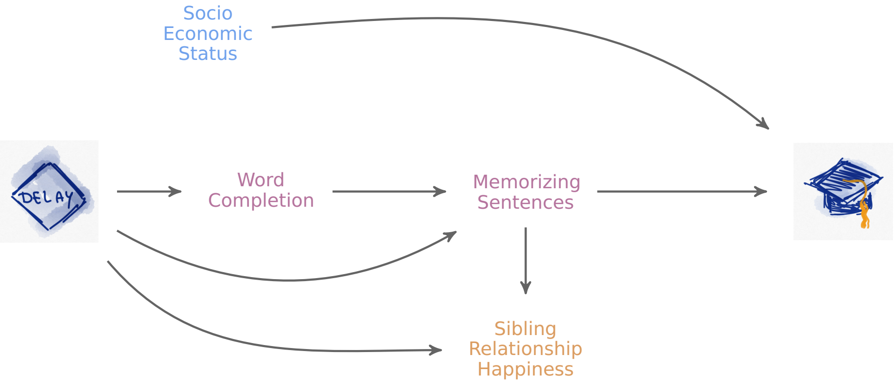
Sibling
Relationship
Happiness

Causal Relationships



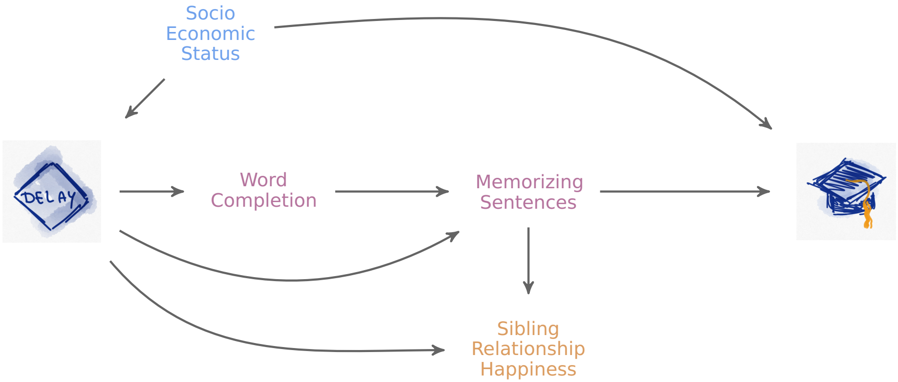
Causal Directed Acyclic Graph (DAG) $\mathcal{D}$.

Interventional DAG



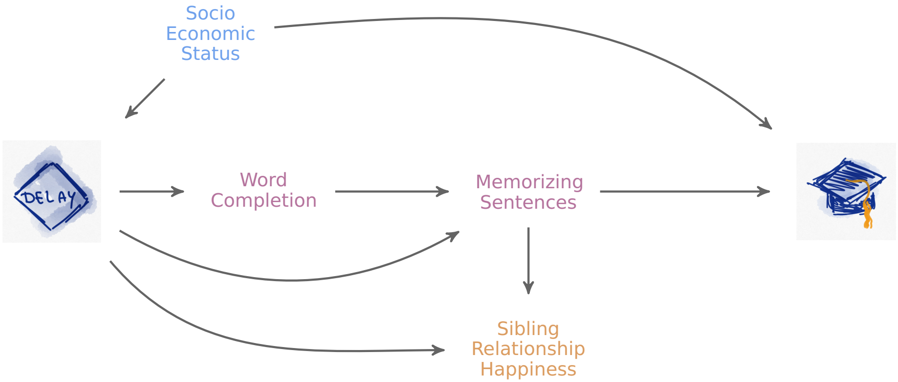
- *Randomized experiment*, e.g: each participant is randomly assigned to treatment or control.
- Any change in response due to a change in treatment goes through *causal paths*.
- $do(x_A)$: an intervention that sets variable X_A to x_A.
- $f(x_Y|do(x_A)) \rightarrow$Causal Effect

Observational Causal DAG



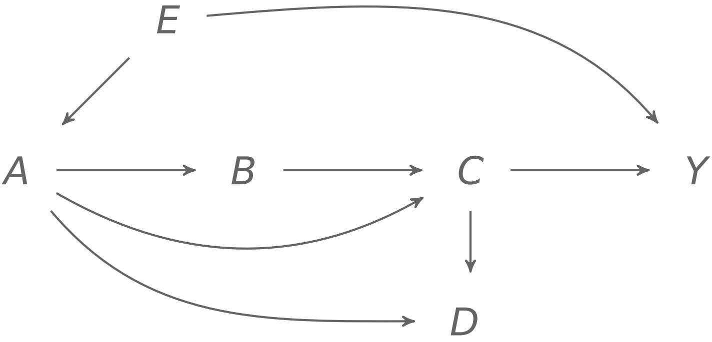
- $f(x_{\mathbf{V}}) \to$ Observational Data
- Access to: $f(x_Y|x_A), f(x_Y), \ldots$
- **Issues**: 1. In general, $f(x_Y|do(x_A)) \neq f(x_Y|x_A)$.

Observational Causal DAG



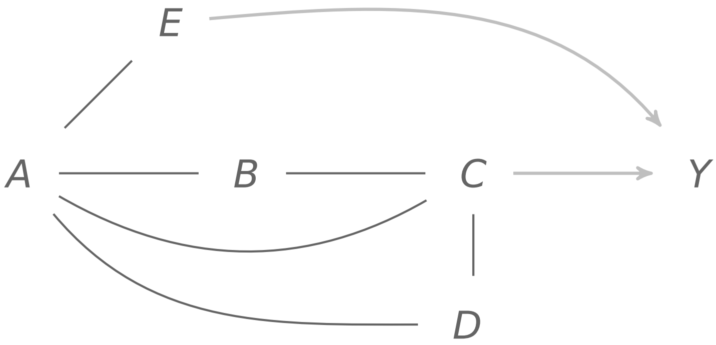
- $f(x_{\mathbf{V}}) \rightarrow$ Observational Data
- Access to: $f(x_Y|x_A), f(x_Y), \ldots$
- **Issues**: 1. In general, $f(x_Y|do(x_A)) \neq f(x_Y|x_A)$. 2. We may not know the full graph.

What if we do not know the DAG?



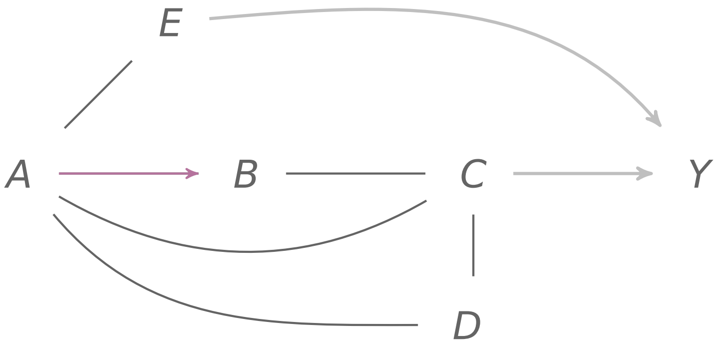
Causal Directed Acyclic Graph (DAG) $\mathcal{D}$.

What if we do not know the DAG?



Completed Partially Directed Acyclic Graph (CPDAG).

What if we do not know the DAG?



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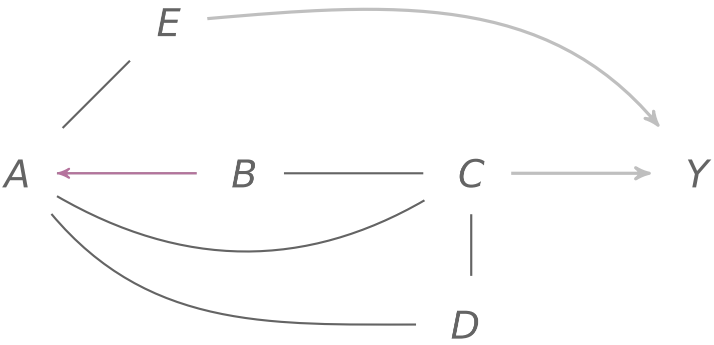

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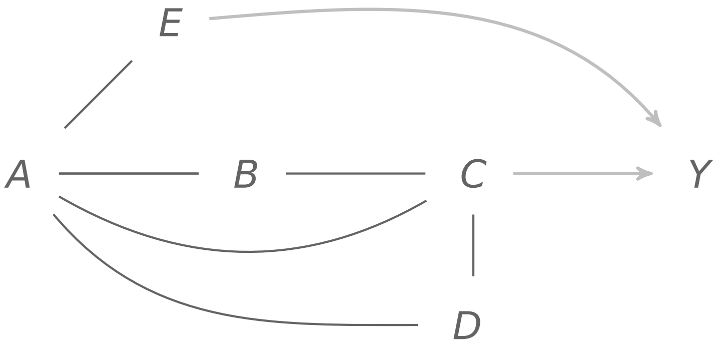

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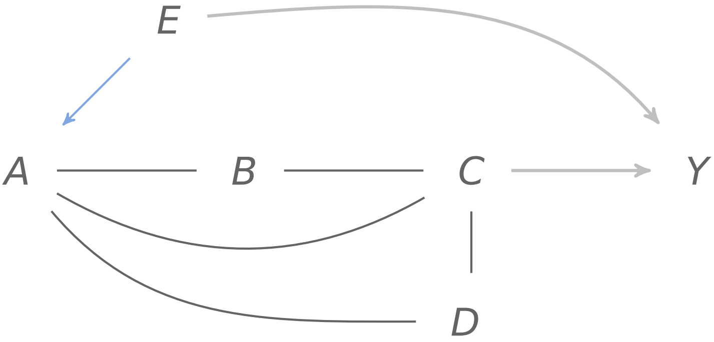
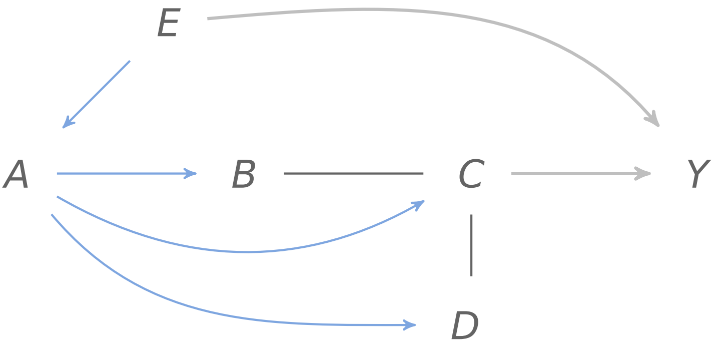


Partially Directed Acyclic Graph (PDAG).

- Expert knowledge of causal relations, previous experiments, model restrictions. . .

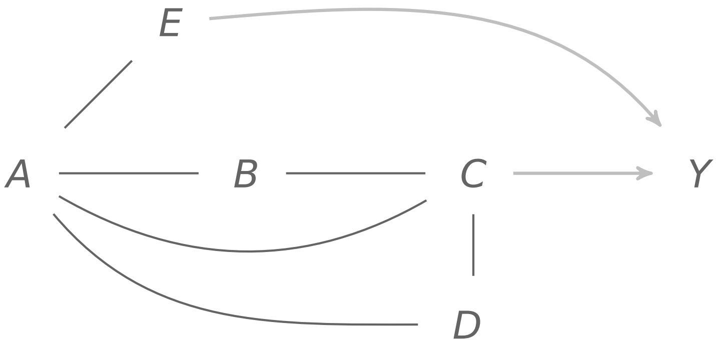
What if we do not know the DAG?



Maximally oriented Partially Directed Acyclic Graph (MPDAG).

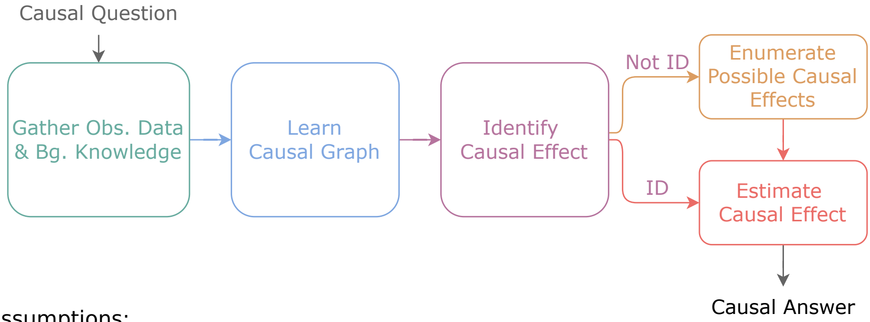
Expert knowledge of causal relations, previous experiments, model restrictions. . .

What if we do not know the DAG?



Completed Partially Directed Acyclic Graph (CPDAG).

Causal Framework



Assumptions:

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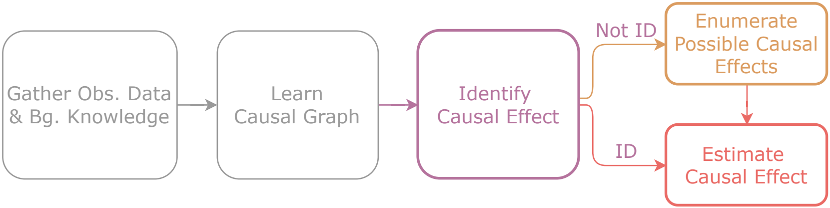
- Do we know all variables that explain or moderate link? Yes.
- Do we know all relationships between these variables? No.

1) Can we uniquely identify the causal effect or a set of possible effects?

2) How strong is this causal relationship?
   - How to construct an estimator?
   - What estimator is optimal in terms of minimal variance?

My Work



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- Perković (2020). Identifying total causal effects in MPDAGs. *UAI 2020*.
- Guo and Perković (2021). Minimal enumeration of all possible total effects in a Markov equivalence class. *AISTATS 2021*.
- Guo and Perković (2022). Efficient Least Squares for Estimating Total Effects under Linearity and Causal Sufficiency. *JMLR*.
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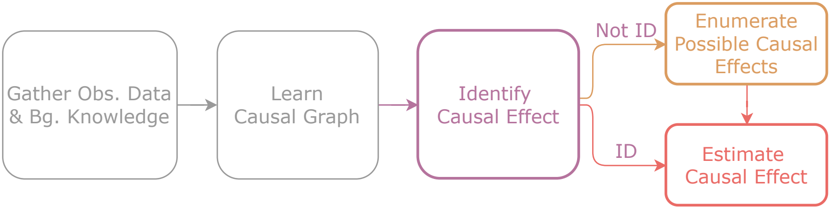
My Work



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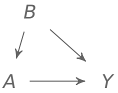
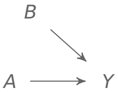
DAGs and Distributions

- Observational density $f(x_{\mathbf{V}})$
- Interventional density $f(x_{\mathbf{V}} | do(x_A))$.

- A DAG $\mathcal{D}$ is causal if for all observational and interventional densities:

$$f(x_{\mathbf{V}}) = \prod_{J \in \mathbf{V}} f(x_J | x_{pa(J)}) \quad \text{and} \quad f(x_{\mathbf{V}} | do(x_A)) = \prod_{J \in \mathbf{V} \setminus \{A\}} f(x_J | x_{pa(J)})$$





$$f(x_B, x_A, x_Y) = f(x_Y | x_B, x_A) f(x_A | x_B) f(x_B)$$

$$f(x_B, x_Y | do(x_A)) = f(x_Y | x_B, x_A) f(x_B)$$

How to define a causal effect?

Total causal effect

- Total causal effect, τ_{AY}, always defined as some function of $f(x_Y|do(X_A = x_A))$, E.g:

$$\tau_{AY} = \mathbb{E}[X_Y|do(X_A = x_A + 1)] - \mathbb{E}[X_Y|do(X_A = x_A)]$$

Identifiability

- A total causal effect is identifiable from observational data and a causal graph if $f(x_Y|do(x_A))$ can be expressed as a function of $f(x_{\mathbf{v}})$.

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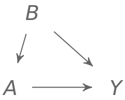
Identifiability

- A total causal effect is identifiable from observational data and a causal graph if

$f(x_Y|do(x_A))$ can be expressed as a function of $f(x_{\mathbf{v}})$.

- Given the causal DAG, every total causal effect is identifiable.



$$\begin{aligned} f(x_Y|do(x_A)) &= \int f(x_B, x_Y|do(x_A))dx_B \\ &= \int f(x_Y|x_B, x_A)f(x_B)dx_B. \end{aligned}$$

G-formula (Robins ’86, Pearl ’93)

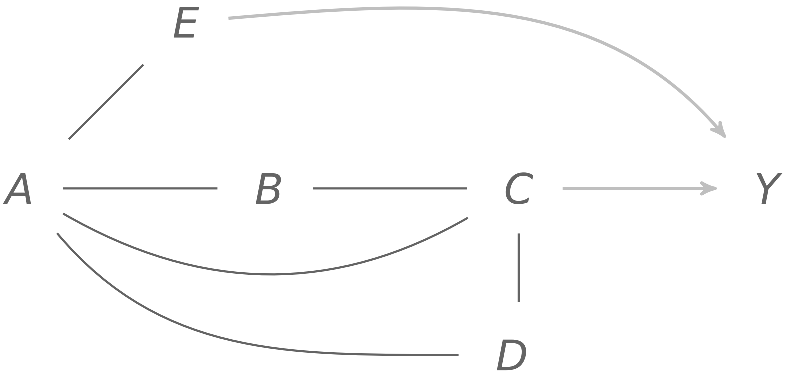
What if we don't know the DAG?

- A causal effect is **not always** identifiable from obs. data and a causal MPDAG.

Graphical criterion	DAG	CPDAG	MPDAG
Adjustment (Pearl '93, Shpitser et al '10)	$\Rightarrow$		
Generalized Adjustment (Perković et al '15, '17, '18)	$\Rightarrow$	$\Rightarrow$	$\Rightarrow$
G-formula, Truncated Factorization (Robins '86, Pearl '93)	$\Leftrightarrow$		
Generalized G-formula (Perković '20)	$\Leftrightarrow$	$\Leftrightarrow$	$\Leftrightarrow$

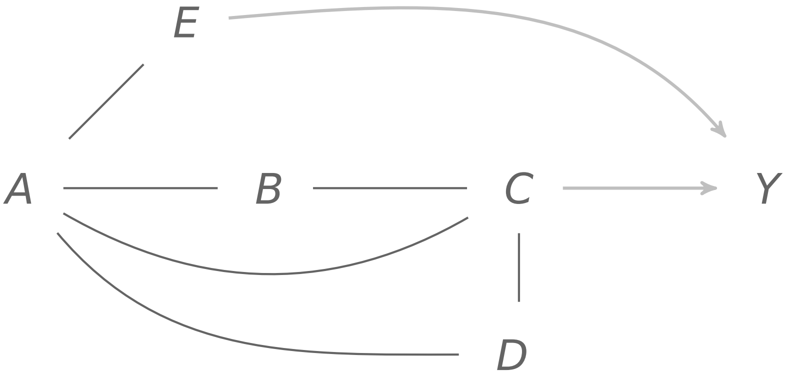
$\Rightarrow$ - sufficient for identification,
$\Leftrightarrow$ - necessary and sufficient for identification

Identifiability Condition



- Can we uniquely identify the effect?

Identifiability Condition

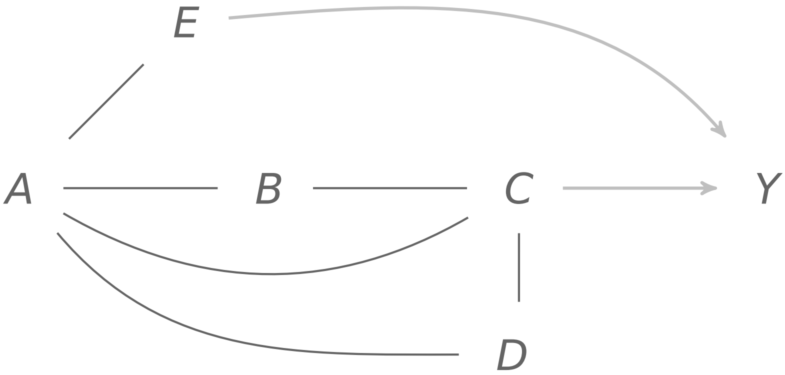


Theorem (Perković, 2020)

The total causal effect of X_A on X_Y is identifiable in MPDAG $\mathcal{G}$ if and only if **all possibly causal paths** from A to Y start with a directed edge in $\mathcal{G}$.

- Can we uniquely identify the effect?

Identifiability Condition



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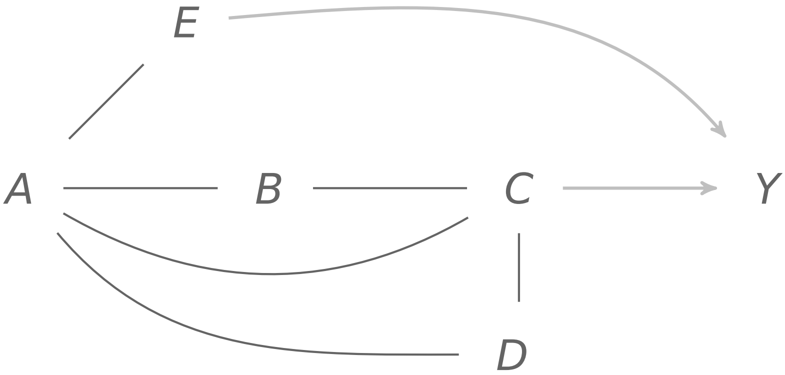
- Can we uniquely identify the effect? **No**.

Identifiability Condition



Theorem (Perković, 2020)

The total causal effect of X_A on X_Y is identifiable in MPDAG $\mathcal{G}$ if and only if **all possibly causal paths** from A to Y start with a directed edge in $\mathcal{G}$.

- Can we uniquely identify the effect? **No.**
- Can we identify the set of possible causal effects? **Yes.**

Set Identification

We want to have a list of possible total effects (**set identification**).

Partition of the equivalence class of DAGs such that **set identification** is

1) **complete**: $f(x_Y|\mathrm{do}(x_A))$ is identifiable under each partition

2) **minimal**: $\mathbb{E}[X_Y|\mathrm{do}(x_A)]$ are distinct functionals of x_A between partitions!

Set Identification

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Partition of the equivalence class of DAGs such that **set identification** is

1) **complete**: $f(x_Y|\mathrm{do}(x_A))$ is identifiable under each partition
   We could enumerate over
   - all DAGs (Maathuis et al, '09)
   - the valid parent sets of A (Maathuis et al, '09, Nandy et al, '17, Perković et al, '17, Witte et al, '20, Fang and He, '20)
   - orientation of $A-$ on possibly causal paths to Y (Liu et al, '20)
2) **minimal**: $\mathbb{E}[X_Y|\mathrm{do}(x_A)]$ are distinct functionals of x_A between partitions!

Theorem (Perković, 2020)

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   - orientation of $A-$ on possibly causal paths to Y (Liu et al, '20)
2) **minimal**: $\mathbb{E}[X_Y|\text{do}(x_A)]$ are distinct functionals of x_A between partitions!
   - None of the above are minimal. Why is Liu et al, 20 not minimal?

Theorem (Perković, 2020)

The total causal effect of X_A on X_Y is identifiable in MPDAG $\mathcal{G}$ if and only if **all possibly causal paths** from A to Y start with a directed edge in $\mathcal{G}$.

Optimal enumeration

Theorem (Perković, 2020)

The total causal effect of X_A on X_Y is identifiable in MPDAG $\mathcal{G}$ if and only if **all possibly causal paths** from A to Y start with a directed edge in $\mathcal{G}$.

Input: MPDAG $\mathcal{G}$, $A, Y \in \mathbf{V}$ and $A \neq Y$.

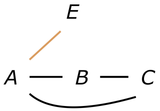
Algorithm FirstTry

1. Pick $A - V_1$ such that there is a possibly causal path $A, V_1, \ldots, Y$.
2. $\mathcal{G}_1 \leftarrow \text{MPDAG}(\mathcal{G}, A \to V_1)$, $\mathcal{G}_2 \leftarrow \text{MPDAG}(\mathcal{G}, A \leftarrow V_1)$
3. Recurse on $\mathcal{G}_1$ and $\mathcal{G}_2$ until $f(x_Y|\text{do}(x_A))$ is identified

MPDAG$(\mathcal{G}, R)$ adds orientations R to $\mathcal{G}$ and completes orientation rules.

Enumeration

Omitted *D* and *Y* for simplicity.



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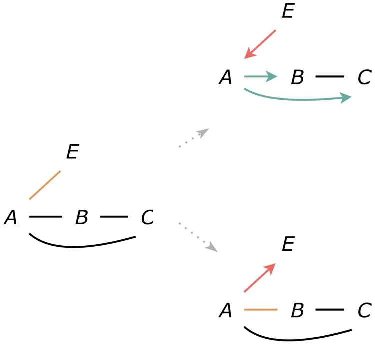

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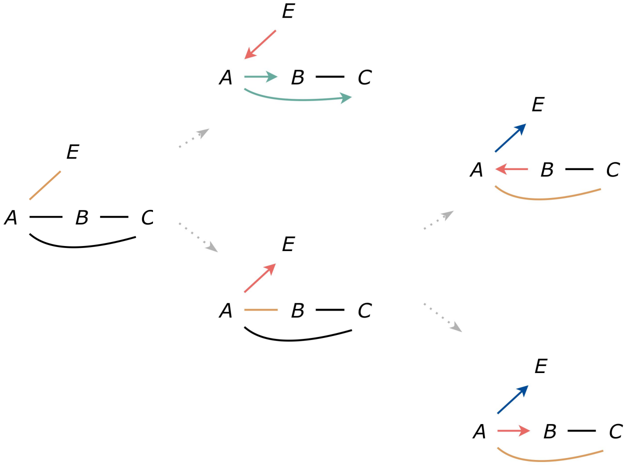

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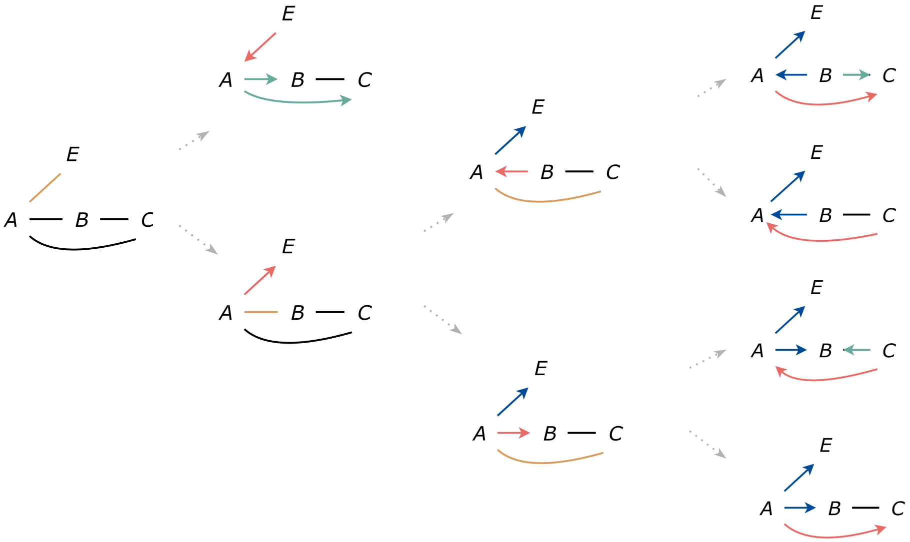

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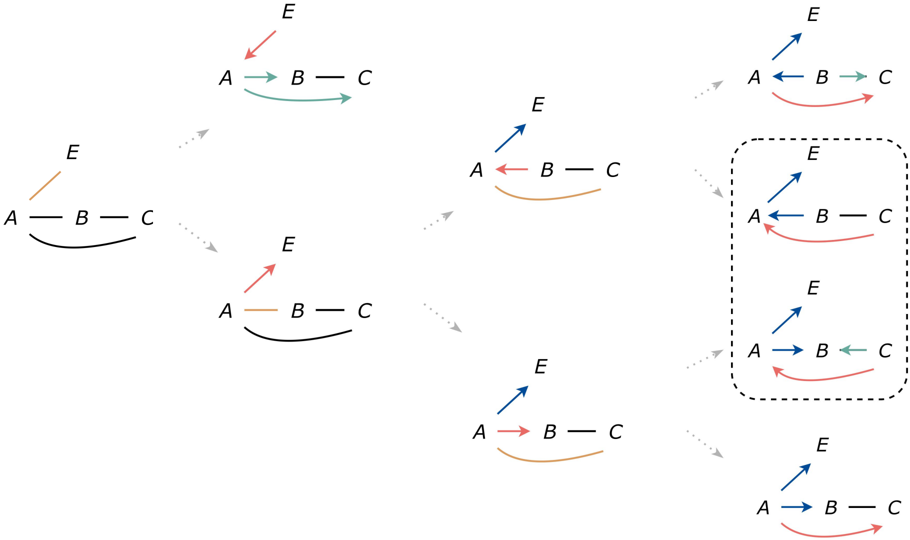

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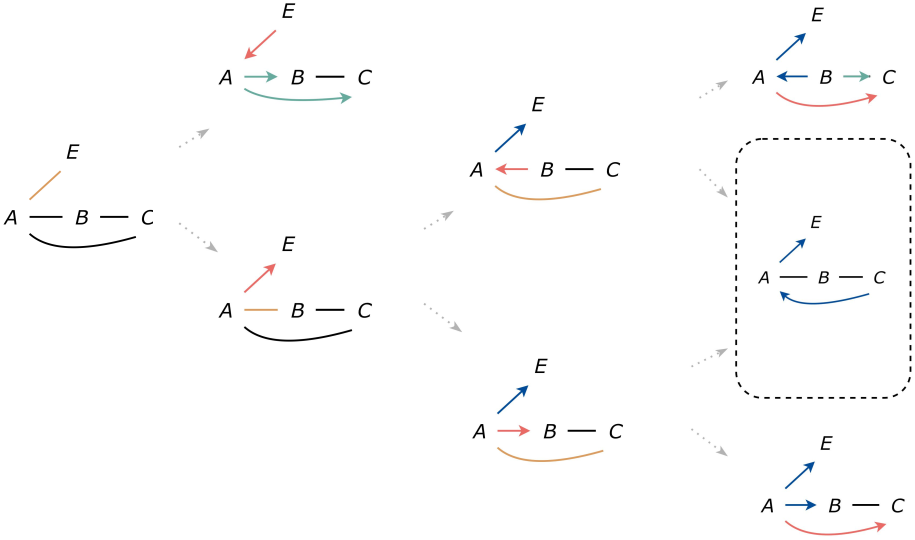
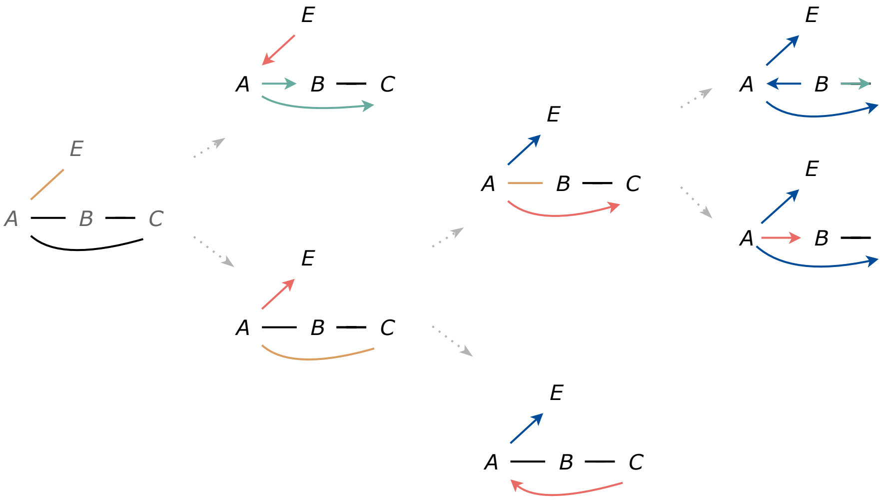


Optimal Enumeration

Orienting $A - E$ then $A - C$ ...

- $A - C$ should be oriented first because the *status* of $A - B - C - Y$ depends on $A - C - Y$.



Optimal Enumeration

Algorithm IDGraphs, (Guo & Perković, 2021)

1. Pick $A - V_1$ such that $A, V_1, \ldots, Y$ is a shortest possibly causal path from A to Y.
2. $\mathcal{G}_1 \leftarrow$ MPDAG($\mathcal{G}, A \rightarrow V_1$), $\mathcal{G}_2 \leftarrow$ MPDAG($\mathcal{G}, A \leftarrow V_1$)
3. Recurse on $\mathcal{G}_1$ and $\mathcal{G}_2$ until identified

Theorem (Guo & Perković, 2021)

$(\mathcal{G}_1, \ldots, \mathcal{G}_m)$ output by the algorithm is **complete** and **minimal**.

- A small change makes a big difference!
- Have a version for the multiple exposure case as well.
- In R package `eff2`.

Marshmallow Test

Assumptions:

- Access to observational data + domain knowledge.
- Do we know all variables that explain or moderate link? Yes.
- Do we know all relationships between these variables? No.

1) Can we uniquely identify the causal effect or a set of possible effects?

   Yes (Perković 2020, Guo & Perković, 2021).

2) How strong is this causal relationship?

   - How to construct an estimator?
   - What estimator is optimal in terms of minimal variance?

Marshmallow Test

Assumptions:

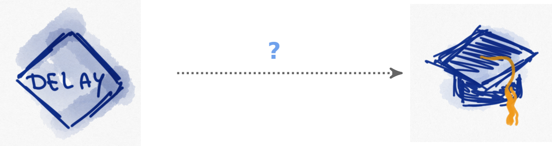
- Access to observational data + domain knowledge.
- Do we know all variables that explain or moderate link? Yes.
- Do we know all relationships between these variables? No.
- Data is generated by a linear structural causal model (SCM).

1) Can we uniquely identify the causal effect or a set of possible effects?

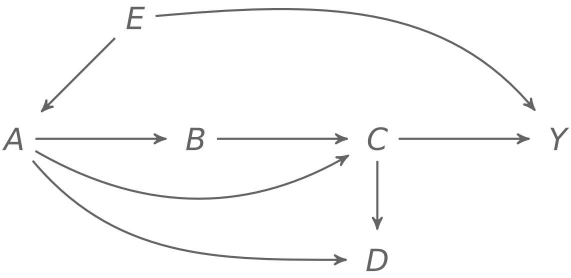
   Yes (Perković 2020, Guo & Perković, 2021).

2) How strong is this causal relationship?

   - How to construct an estimator?
   - What estimator is optimal in terms of minimal variance?



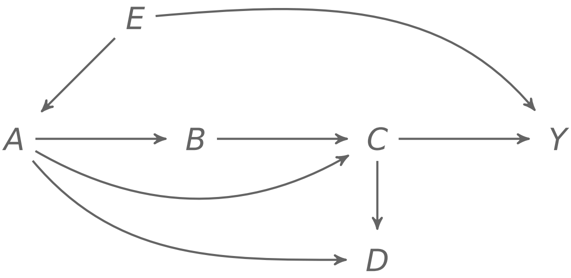
Causal DAG, Linear Structural Causal Model (SCM)



- Data is generated by:

$$
\begin{aligned}
X_E &= \epsilon_E \\
X_A &= \gamma_{EA} X_E + \epsilon_A \\
X_B &= \gamma_{AB} X_A + \epsilon_B \\
X_C &= \gamma_{AC} X_A + \gamma_{BC} X_B + \epsilon_C \\
X_D &= \gamma_{AD} X_A + \gamma_{CD} X_C + \epsilon_D \\
X_Y &= \gamma_{BY} X_B + \gamma_{CY} X_C + \gamma_{EY} X_E + \epsilon_Y \\
\mathbb{E}\,\epsilon &= 0, \quad 0 < \operatorname{var} \epsilon_i < \infty, \quad \epsilon_i \text{ are mutually independent,}
\end{aligned}
$$

Causal DAG, Linear Structural Causal Model (SCM)



- Data is generated by:

$$X = \Gamma^\intercal X + \epsilon, \qquad \Gamma = (\gamma_{ij}), \quad I \not\to J \Rightarrow \gamma_{ij} = 0,$$

$$\mathbb{E}\,\epsilon = 0, \quad 0 < \operatorname{var}\epsilon_i < \infty, \quad \epsilon_i \text{ are mutually independent,}$$

$$\Gamma \text{ is the weighted adjacency matrix.}$$

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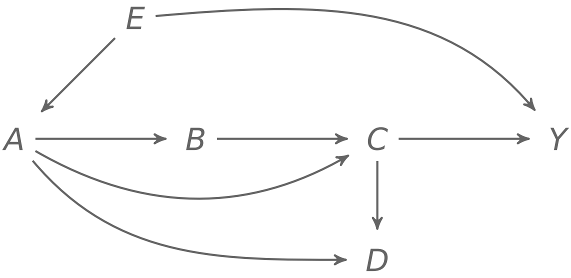

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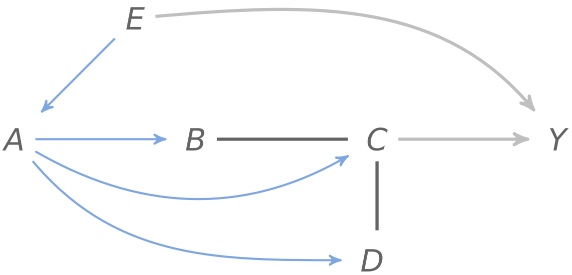
Γ is the weighted adjacency matrix.

- By the path tracing rules (Wright, 1934) and the G-formula:

$$\tau_{AY} = \cdots = \gamma_{ac}\gamma_{cy} + \gamma_{ab}\gamma_{bc}\gamma_{cy}.$$

Block-recursive Reparametrization



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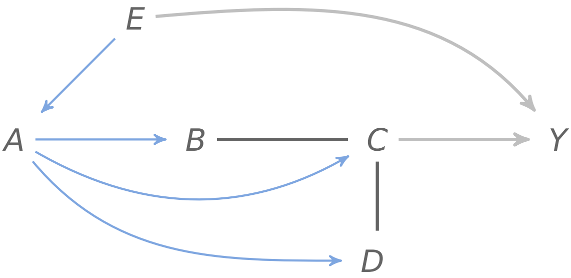
$$\mathbb{E}\,\epsilon = 0, \quad 0 < \operatorname{var}\epsilon_I < \infty, \quad \epsilon_I \text{ are mutually independent.}$$

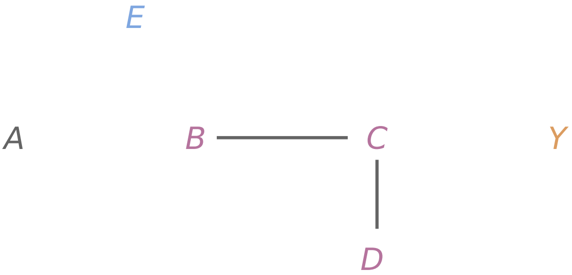
- **Problem:** Γ is **not uniquely identified**.

Block-recursive Reparametrization



- **Idea**: Consider **buckets** (maximal undirected connected components) in $\mathcal{G}$:

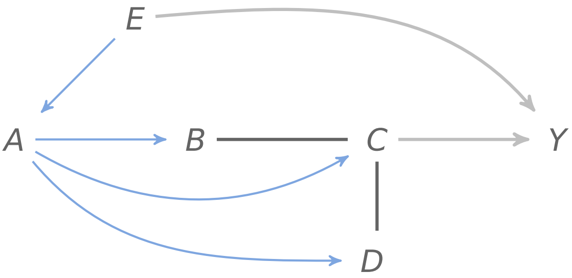
Block-recursive Reparametrization



- **Idea**: Consider **buckets** (maximal undirected connected components) in $\mathcal{G}$:

$$\mathbf{B_1} = \{E\},\ \mathbf{B_2} = \{A\},\ \mathbf{B_3} = \{B, C, D\},\ \mathbf{B_4} = \{Y\}.$$

Block-recursive Reparametrization



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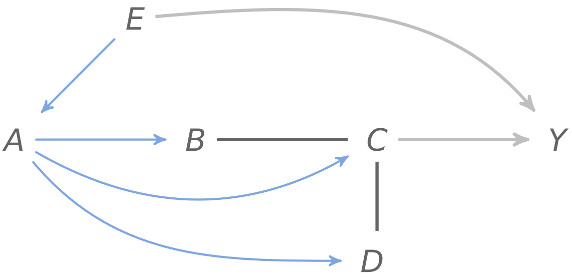
  1. The “between bucket” causal effects are **identifiable**. (Perković 2020).
  2. **Restrictive property:** Each node in a bucket has the same out-of-bucket parents (Guo and Perković, 2022).

- We use this to reparametrize the SCM.

Block-recursive Reparametrization



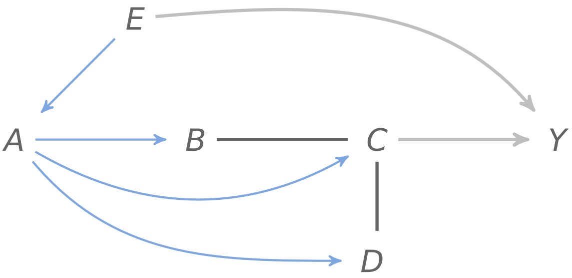
- **Idea**: Consider **buckets** (maximal undirected connected components) in $\mathcal{G}$:

$$\mathbf{B_1} = \{E\},\ \mathbf{B_2} = \{A\},\ \mathbf{B_3} = \{B, C, D\},\ \mathbf{B_4} = \{Y\}.$$

$$X_{\mathbf{B_i}} = \Gamma^{\top}_{\mathrm{pa}(\mathbf{B_i},\mathcal{G}),\mathbf{B_i}} X_{\mathrm{pa}(\mathbf{B_i},\mathcal{G})} + \Gamma^{\top}_{\mathbf{B_i}} X_{\mathbf{B_i}} + \epsilon_{\mathbf{B_i}},$$

,

Block-recursive Reparametrization



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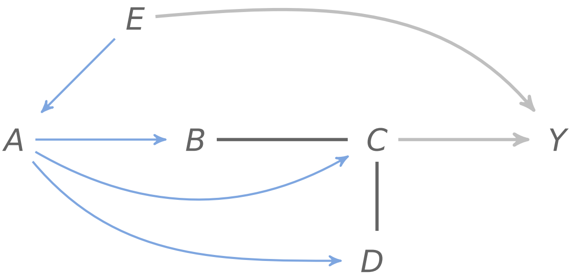
$$\begin{aligned} X_{\mathbf{B_i}} &= \left(I - \Gamma_{\mathbf{B_i}}\right)^{-\top} \Gamma^{\top}_{\mathrm{pa}(\mathbf{B_i},\mathcal{G}),\mathbf{B_i}} X_{\mathrm{pa}(\mathbf{B_i},\mathcal{G})} + \left(I - \Gamma_{\mathbf{B_i}}\right)^{-\top} \epsilon_{\mathbf{B_i}} \\ &= \Lambda^{\top}_{\mathrm{pa}(\mathbf{B_i},\mathcal{G}),\mathbf{B_i}} X_{\mathrm{pa}(\mathbf{B_i},\mathcal{G})} + \varepsilon_{\mathbf{B_i}}, \end{aligned}$$

Block-recursive Reparametrization



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$$\mathbf{B_1} = \{E\},\ \mathbf{B_2} = \{A\},\ \mathbf{B_3} = \{B, C, D\},\ \mathbf{B_4} = \{Y\}.$$

$$X_{\mathbf{B_i}} = \Gamma^{\top}_{\mathrm{pa}(\mathbf{B_i},\mathcal{G}),\mathbf{B_i}} X_{\mathrm{pa}(\mathbf{B_i},\mathcal{G})} + \Gamma^{\top}_{\mathbf{B_i}} X_{\mathbf{B_i}} + \epsilon_{\mathbf{B_i}},$$

$$X_{\mathbf{B_i}} = \left(I - \Gamma_{\mathbf{B_i}}\right)^{-\top} \Gamma^{\top}_{\mathrm{pa}(\mathbf{B_i},\mathcal{G}),\mathbf{B_i}} X_{\mathrm{pa}(\mathbf{B_i},\mathcal{G})} + \left(I - \Gamma_{\mathbf{B_i}}\right)^{-\top} \epsilon_{\mathbf{B_i}}$$
$$= \Lambda^{\top}_{\mathrm{pa}(\mathbf{B_i},\mathcal{G}),\mathbf{B_i}} X_{\mathrm{pa}(\mathbf{B_i},\mathcal{G})} + \varepsilon_{\mathbf{B_i}},$$

- Suggests re-writing τ_{AY} using elements of Λ and estimating $\Lambda_{\mathrm{pa}(\mathbf{B_i},\mathcal{G}),\mathbf{B_i}}$ using least squares coefficients from $\mathbf{B_i} \sim \mathrm{pa}(\mathbf{B_i}, \mathcal{G})$ $\rightarrow$ $\mathcal{G}$-regression.

Efficiency

Theorem ($\mathcal{G}$-regression, Guo and Perković, 2022)

Suppose τ_{AY} is identifiable given MPDAG $\mathcal{G}$ and let

$\hat{\tau}^{\mathcal{G}}_{AY}$ be the **$\mathcal{G}$-regression estimator**.

Then for **any consistent estimator** $\hat{\tau}_{AY}$ of τ_{AY} such that

$\hat{\tau}_{AY}$ is a **differentiable function of the sample covariance**

it holds that

$$\mathrm{avar}\left(\hat{\tau}_{AY}\right) \geq \mathrm{avar}\left(\hat{\tau}^{\mathcal{G}}_{AY}\right),$$

avar - asymptotic variance.

This includes estimators based on:

- covariate adjustment (Henckel et al, 2022, Witte et al, 2020),
- recursive regressions (Nandy et al, 2017, Gupta et al, 2020),
- modified Cholesky decomposition (Nandy et al, 2017).

Marshmallow Test

Assumptions:

- Access to observational data + domain knowledge.
- Do we know all variables that explain or moderate link? Yes.
- Do we know all relationships between these variables? No.
- Data is generated by a linear structural causal model (SCM).

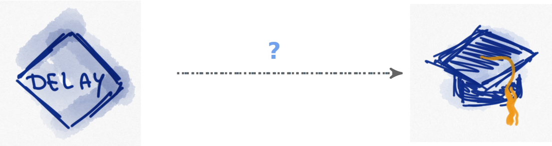
1) Can we uniquely identify the causal effect or a set of possible effects?

   Yes (Perković 2020, Guo & Perković, 2021).

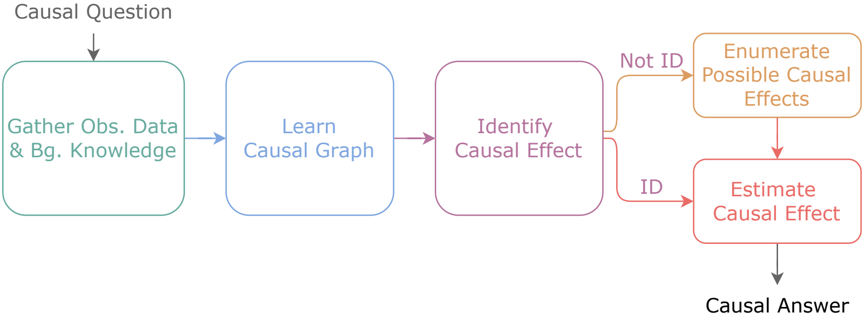
2) How strong is this causal relationship?

   - How to construct an estimator? **Generalized G-Formula** (Perković 2020, Guo & Perković, 2022, Guo, Perković, & Rotnitzky (2022)).

   - What estimator is optimal in terms of minimal variance? **$\mathcal{G}$-regression** (Guo & Perković, 2022).



Causal Framework



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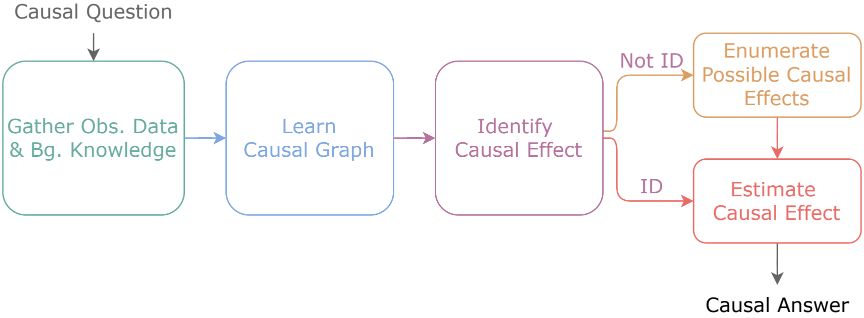
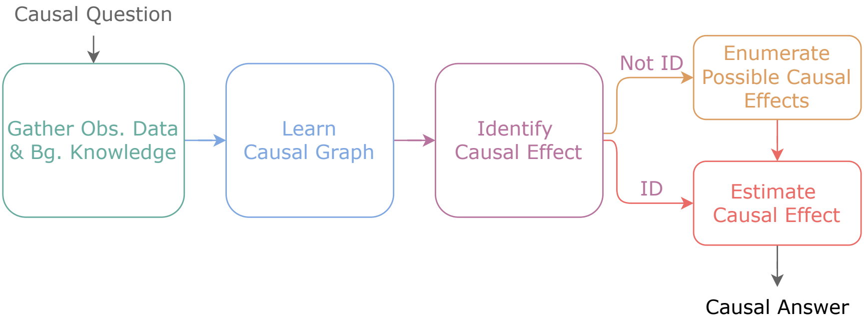


Assumptions:

- Access to observational data + domain knowledge.
- Do we know all variables that explain or moderate link? No. → **Many open problems**.
- Do we know all relationships between these variables? No.

Causal Framework



Assumptions:

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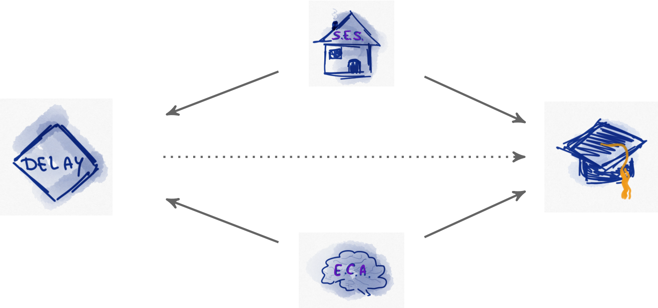
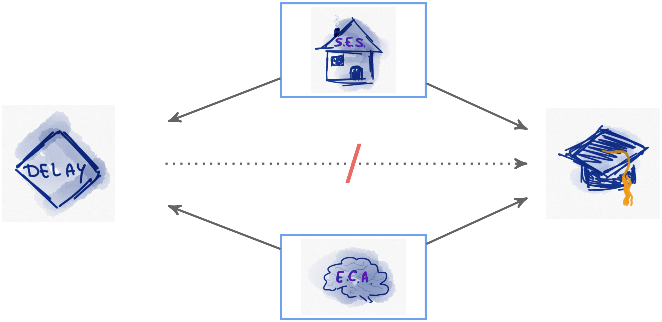
Thanks!

Marshmallow Test Revisited



- Watts, T.W., Duncan, G.J., and Quan, H. (2018) in *Psychological science.*

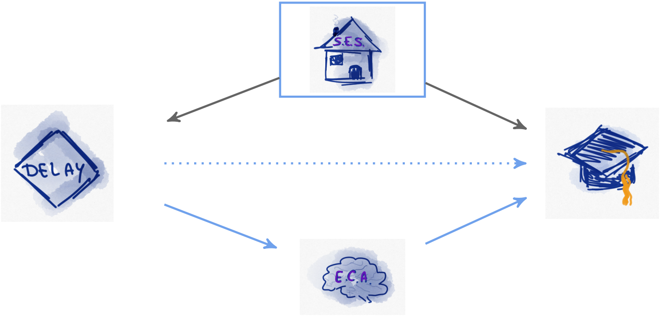
Marshmallow Test Revisited



- Watts, T.W., Duncan, G.J., and Quan, H. (2018) in *Psychological science.*

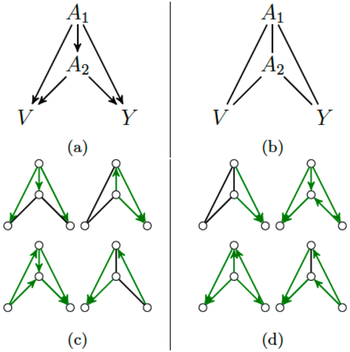
→ "...Associations between delay time and measures of behavioral outcomes at age 15 were much smaller and rarely statistically significant."

Marshmallow Test Re-Revisited



- Doebel, S., Michaelson, L.E., and Munakata, Y. (2019), *Psychological Science.*
- Falk, A., Kosse, F., and Pinger, P. (2019), *Psychological Science.*
- Watts, T.W., and Duncan, G.J. (2019), *Psychological Science.*
- Benjamin, D.J., Laibson D., **Mischel, W.**, Peake, P.K., Shoda, Y., Wellsjo, A.S., and Wilson N.W. (2020), *Journal of Economic Behavior & Organization*

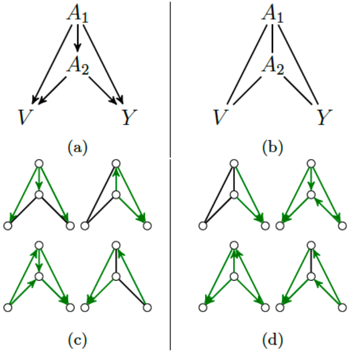
Simulation results



	A_1 on Y (c)
true effect	3
true poss. effects	$\{3, 2, 1.8, 0\}$
our method	$\{2.9, 2.1, 1.9, 0\}$
IDA (optimal)	$\{2.9, (2.1)^2, 1.9, 0\}$
IDA (local, collapsible)	$\{2.9, 2.1, 2.2, 1.9, 0\}$
joint-IDA	—

- Generated with a linear structural causal model with Gaussian errors and $n = 100$.
- $(a)^b$ denotes that a appears with multiplicity b.

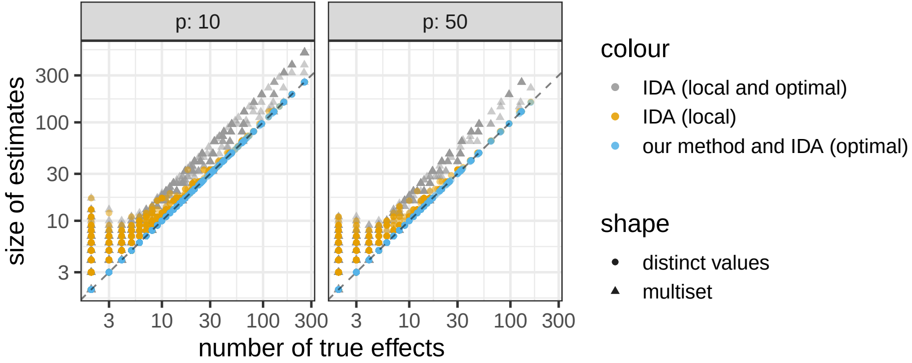
Simulation results



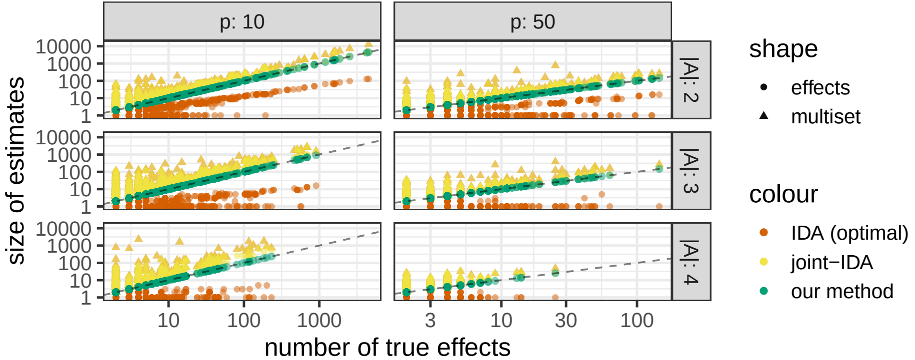
	A_1, A_2 on Y (d)
true effect	(2,1)
true poss. effects	$\{(2,1), (3,0), (0,2), (0,0)\}$
our method	$\{(2.1, 0.9), (2.9, 0), (0, 1.9), (0, 0)\}$
IDA (optimal)	$\{(2.1, 0.9)^6, (0,0)^2, (\texttt{NA}, \texttt{NA})^2\}$
IDA (local, collapsible)	—
joint-IDA	$\{(2.1, 0.9)^2, (2.2, 0.9), (1.9, 1.1),$ $(2.2, 1.1)^2, (0, 1.9), (2.9, 0), (0, 0)^2\}$

- Generated with a linear structural causal model with Gaussian errors and $n = 100$.
- $(a)^b$ denotes that a appears with multiplicity b.

Simulation: size of possible effects



Simulation: size of possible effects



Overview

	Comp. Cost	$\lvert A\rvert = 1$	$\lvert A\rvert > 1$	Duplicates
Naive - Enumerate all DAGs:				
global IDA (Maathuis et al, 2009)	$\mathcal{O}(\lvert V\rvert !)$	✓	-	Yes
global joint IDA (Nandy et al, 2017)	$\mathcal{O}(\lvert V\rvert !)$	✓	✓	Yes
Enumerate valid parent sets of A:				
local IDA (Maathuis et al, 2009, Fang & He, 2020)	$\mathcal{O}(2^{l(\mathcal{G})})$	✓	-	Yes
semi-local IDA, joint IDA (P. et al, 2017,Nandy et al, 2017)	$\mathcal{O}(2^{l(\mathcal{G})} poly(\lvert V\rvert))$	✓	✓	Yes
optimal IDA (Witte et al, 2020)	$\mathcal{O}(2^{l(\mathcal{G})} poly(\lvert V\rvert))$	✓	$\sim$	No
Enum. $A-$ on poss. causal paths to Y:				
collapsible IDA (Liu et. al, 2020)	$\mathcal{O}((\lvert V\rvert + \lvert E\rvert)2^{r(\mathcal{G})})$	✓	-	Yes
Recursively enum. over shortest problem paths				

- $l(\mathcal{G})$ - # of undirected edges connected to A
- $r(\mathcal{G})$ - # of edges $A-$ on possibly causal paths to Y, $r(\mathcal{G}) \leq l(\mathcal{G})$

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	Comp. Cost	$\lvert A\rvert = 1$	$\lvert A\rvert > 1$	Duplicates
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Enumerate valid parent sets of A:				
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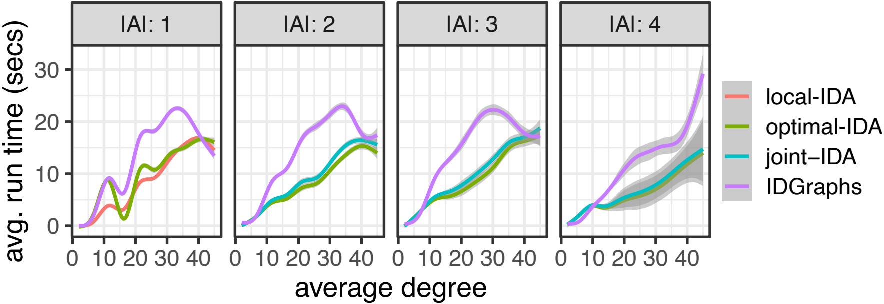
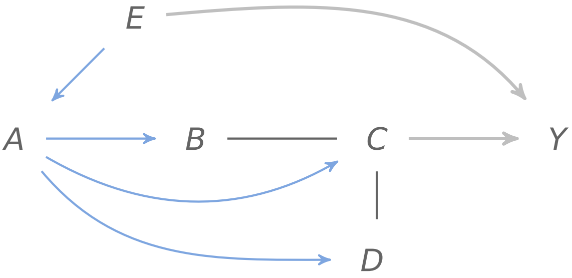
Enum. $A-$ on poss. causal paths to Y:				
collapsible IDA (Liu et. al, 2020)	$\mathcal{O}((\lvert V\rvert + \lvert E\rvert)2^{r(\mathcal{G})})$	✓	-	Yes
Recursively enum. over shortest problem paths				
`IDGraphs` (Guo & Perković)	$\mathcal{O}(2^{m(\mathcal{G})}poly(\lvert V\rvert))$	✓	✓	No

- $l(\mathcal{G})$ - # of undirected edges connected to A
- $r(\mathcal{G})$ - # of edges $A-$ on possibly causal paths to Y, $r(\mathcal{G}) \leq l(\mathcal{G})$
- $m(\mathcal{G})$ - # of recursively id. edges $A-$ on proper possibly causal paths to Y, $m(\mathcal{G}) \leq r(\mathcal{G})$

Average runtime simulation comparison



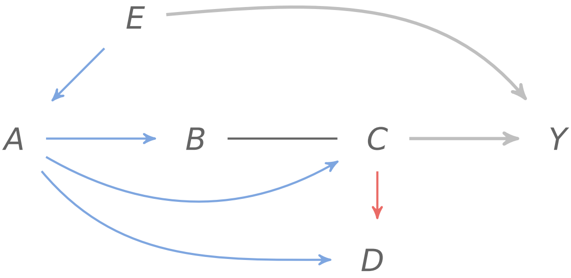
Generalized G-Formula and $\mathcal{G}$-Regression



- Generalized G-Formula and $\mathcal{G}$-regression:

$$\mathbb{E}[X_Y|\mathrm{do}(x_A)] = \int \mathbb{E}[X_Y|x_B, x_C, x_E] f(x_B, x_C|x_A) f(x_E) dx_B dx_C dx_E$$

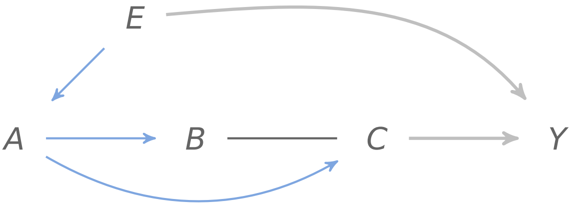
Same Generalized G-Formula and $\mathcal{G}$-Regression



- The generalized G-formula is the same in the above MPDAG.

$$\mathbb{E}[X_Y|\text{do}(x_A)] = \int \mathbb{E}[X_Y|x_B, x_C, x_E] f(x_B, x_C|x_A) f(x_E) dx_B dx_C dx_E$$

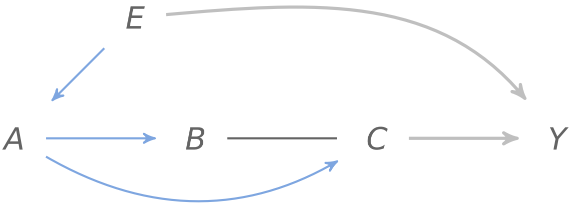
Same Generalized G-Formula and $\mathcal{G}$-Regression



- As well as in the above MPDAG.

$$\mathbb{E}[X_Y|\mathrm{do}(x_A)] = \int \mathbb{E}[X_Y|x_B, x_C, x_E]f(x_B, x_C|x_A)f(x_E)dx_B dx_C dx_E$$

Same Generalized G-Formula and $\mathcal{G}$-Regression



- As well as in the above MPDAG.

$$\mathbb{E}[X_Y|\mathrm{do}(x_A)] = \int \mathbb{E}[X_Y|x_B, x_C, x_E]f(x_B, x_C|x_A)f(x_E)dx_B dx_C dx_E$$

- Indicating that: **measurement** of all variables not needed for efficient causal estimation.
- We explore these implications in Guo, Perković and Rotnitzky (2022). Opportunities for future work.

Block-recursive reparametrization

Proposition (Block-recursive form, Guo and Perković, 2022)

Let $\mathbf{B_1}, \ldots, \mathbf{B_K}$ be the ordered bucket decomposition of $\mathbf{V}$ in MPDAG $\mathcal{G}$. Then

$$X = \Lambda^\intercal X + \varepsilon, \qquad \Lambda = (\lambda_{ij}),\ J \in \mathbf{B_k},\ I \notin \mathrm{pa}(\mathbf{B_k}, \mathcal{G}) \quad \Rightarrow \quad \lambda_{ij} = 0,$$
$$\mathbb{E}\,\varepsilon = 0, \quad \mathbb{E}\,\varepsilon_{\mathbf{B_k}} \varepsilon_{\mathbf{B_k}}^\intercal \succ \mathbf{0}, \quad \varepsilon_{\mathbf{B_k}} \text{ mutually independent},$$

Two nice things happen under this re-parametrization:

- For $\mathbf{S} = \mathrm{An}(Y, \mathcal{G}_{\mathbf{V} \setminus \{A\}})$, τ_{AY} can be identified as

$$\tau_{AY} = \Lambda_{A,\mathbf{S}} \left[(I - \Lambda_{\mathbf{S},\mathbf{S}})^{-1} \right]_{\mathbf{S},Y}.$$

  The bucket-wise error distribution is a nuisance.
- Under Gaussian errors, the MLE for each $\Lambda_{\mathrm{pa}(\mathbf{B_i}, \mathcal{G}), \mathbf{B_i}}$ corresponds to the least squares coefficients from $\mathbf{B_i} \sim \mathrm{pa}(\mathbf{B_i}, \mathcal{G})$. $\rightarrow$ $\mathcal{G}$-regression.

Efficiency

Theorem ($\mathcal{G}$-regression, Guo and Perković, 2022)

If τ_{AY} is identifiable given MPDAG $\mathcal{G}$, the **$\mathcal{G}$-regression estimator** is defined as:

$$\hat{\tau}^{\mathcal{G}}_{AY} := \hat{\Lambda}^{\mathcal{G}}_{A,\mathbf{S}} \left[(I - \hat{\Lambda}^{\mathcal{G}}_{\mathbf{S},\mathbf{S}})^{-1} \right]_{\mathbf{S},Y},$$

where $\mathbf{S} = \mathrm{An}(Y, \mathcal{G}_{\mathbf{V}\setminus\{A\}})$, and $\hat{\Lambda}^{\mathcal{G}}$ is matrix consisting of least squares coefficients for each "bucket" regression.

Then for **any consistent estimator** $\hat{\tau}_{AY}$ of τ_{AY} such that $\hat{\tau}_{AY}$ is a **differentiable function of the sample covariance** it holds that

$$\mathrm{avar}\left(\hat{\tau}_{AY}\right) \geq \mathrm{avar}\left(\hat{\tau}^{\mathcal{G}}_{AY}\right),$$

avar - asymptotic variance.

This includes estimators based on:

- covariate adjustment (Henckel et al, 2022, Witte et al, 2020),
- recursive regressions (Nandy et al, 2017, Gupta et al, 2020),
- modified Cholesky decomposition (Nandy et al, 2017).

Simulation results

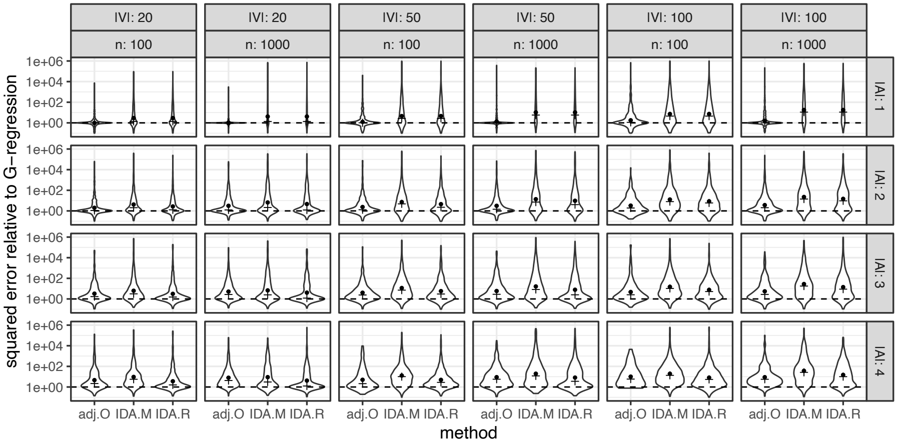
An instance is simulated by the following steps.

1. Draw $\mathcal{D}$ from a random graph ensemble.
2. Take $\mathcal{G} = \text{CPDAG}(\mathcal{D})$.
3. Simulate data from a linear SCM with random error type (normal, t, logistic, uniform).
4. Choose (A, Y) such that τ_{AY} is identified from $\mathcal{G}$.
5. Compute squared error $err = \|\tau_{AY} - \hat{\tau}_{AY}\|^2$.

We compare $\mathcal{G}$-regression to the following estimators:

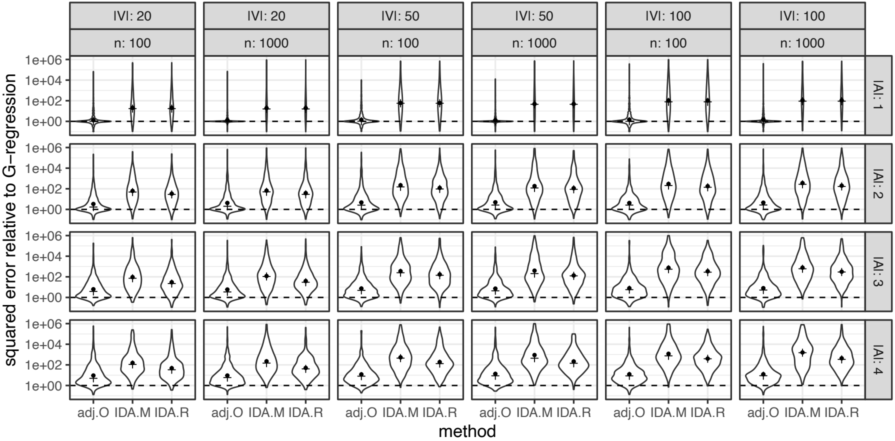
- `adj.O`: optimal adjustment estimator (Henckel et al, 2022), or
- `IDA.M`: joint-IDA estimator based on modifying Cholesky decompositions (Nandy et al, 2017), or
- `IDA.R`: joint-IDA estimator based on recursive regressions (Nandy et al, 2017).

Simulation results



Violin plots displaying relative squared errors $\frac{estimator.err}{\mathcal{G}-reg.err}$ given GES estimated CPDAG.

Simulation results



Violin plots displaying relative squared errors $\frac{estimator.err}{\mathcal{G}-reg.err}$ given the true DAG.

Simulation results

Table: Percentage of identified instances not estimable using contending estimators. All instances are estimable with $\mathcal{G}$-regression.

Estimator	$\|\mathbf{A}\|$	$\|\mathbf{V}\| = 20$	$\|\mathbf{V}\| = 50$	$\|\mathbf{V}\| = 100$
`adj.O`	1	0%	0%	0%
	2	17%	10%	5%
	3	30%	18%	15%
	4	36%	29%	22%
`IDA.M`	1	29%	32%	32%
	2	47%	51%	50%
	3	61%	59%	63%
	4	72%	69%	71%
`IDA.R`	1	29%	32%	32%
	2	47%	51%	50%
	3	61%	59%	63%
	4	72%	69%	71%

Simulation results

Table: Geometric average of squared errors relative to $\mathcal{G}$-regression, computed from estimable instances.

$\lvert\mathbf{A}\rvert$	$\lvert\mathbf{V}\rvert = 20$ $n = 100$	$\lvert\mathbf{V}\rvert = 20$ $n = 1000$	$\lvert\mathbf{V}\rvert = 50$ $n = 100$	$\lvert\mathbf{V}\rvert = 50$ $n = 1000$	$\lvert\mathbf{V}\rvert = 100$ $n = 100$	$\lvert\mathbf{V}\rvert = 100$ $n = 1000$
`adj.O`						
1	1.3	1.3	1.4	1.3	1.5	1.5
2	3.4	4.2	4.7	4.9	4.2	4.5
3	6.3	5.9	7.4	7.2	7.8	8.0
4	9.3	9.3	12	14	12	12
`IDA.M`						
1	20	19	61	48	103	108
2	62	65	220	182	293	356
3	93	119	354	396	749	771
4	154	222	533	895	1188	1604
`IDA.R`						
1	20	19	61	48	103	108
2	33	38	121	113	176	199
3	30	39	171	135	342	312
4	48	50	187	214	405	432

Simulation results

Table: Geometric average of squared errors relative to $\mathcal{G}$-regression, computed from estimable instances given GES estimated CPDAG

	$\lvert\mathbf{V}\rvert = 20$		$\lvert\mathbf{V}\rvert = 50$		$\lvert\mathbf{V}\rvert = 100$	
$\lvert\mathbf{A}\rvert$	$n = 100$	$n = 1000$	$n = 100$	$n = 1000$	$n = 100$	$n = 1000$
`adj.O`						
1	1.0	1.0	1.2	1.3	1.8	1.6
2	2.0	3.1	2.4	3.1	3.2	3.7
3	3.3	5.2	4.0	5.9	4.7	5.5
4	4.6	7.9	5.0	9.0	10	8.9
`IDA.M`						
5	2.9	4.1	4.5	10	7.3	18
6	4.2	6.6	7.3	14	13	22
7	6.2	6.8	12	16	15	28
8	9.5	9.0	13	20	19	37
`IDA.R`						
9	2.9	4.1	4.5	10	7.3	18
10	2.7	4.6	4.5	9.6	8.5	15
11	3.1	4.1	5.8	7.8	7.6	14
12	3.6	4.2	4.9	8.2	8.1	15

Identification of total causal effect

$\mathbf{S_1}, \ldots, \mathbf{S_K}$ is a partition of $\mathbf{S} = An(Y, \mathcal{G}_{\mathbf{V}\setminus\{A\}})$ induced by $\mathbf{B_1}, \ldots, \mathbf{B_K}$.
Let $\mathbf{F_k} = \{A\} \cap \mathrm{pa}(\mathbf{S_k}, \mathcal{G})$, for all $k \in \{1, \ldots, k\}$. Then

$$P(X_\mathbf{S}|\mathrm{do}(x_A)) = \prod_{k=1}^{K} P(X_{\mathbf{S_k}}|X_{\mathrm{pa}(\mathbf{S_k},\mathcal{G})}) = \prod_{k=1}^{K} P(X_{\mathbf{S_k}}|X_{\mathrm{pa}(\mathbf{S_k},\mathcal{G})\setminus\mathbf{F_k}}, X_{\mathbf{F_k}} = x_{\mathbf{F_k}}),$$

where $x_{\mathbf{F_k}}$ is fixed by the $\mathrm{do}(x_A)$ operation.

$$\begin{aligned} X_{\mathbf{S_k}} \mid \left\{X_{\mathrm{pa}(\mathbf{S_k},\mathcal{G})\setminus\mathbf{F_k}}, X_{F_i} = x_{\mathbf{F_k}}\right\} &=_d \Lambda^{\mathsf{T}}_{\mathrm{pa}(\mathbf{S_k},\mathcal{G})\setminus\mathbf{F_k},\mathbf{S_k}} X_{\mathrm{pa}(\mathbf{S_k},\mathcal{G})\setminus\mathbf{F_k}} + \Lambda_{\mathbf{F_k},\mathbf{S_k}} x_{\mathbf{F_k}} + \varepsilon_{\mathbf{S_k}} \\ &= \Lambda^{\mathsf{T}}_{\mathrm{pa}(\mathbf{S_k},\mathcal{G})\cap\mathbf{S},\mathbf{S_k}} X_{\mathrm{pa}(\mathbf{S_k},\mathcal{G})\cap\mathbf{S}} + \Lambda_{\mathrm{pa}(\mathbf{S_k},\mathcal{G})\cap\{A\},\mathbf{S_k}} x_{\mathrm{pa}(\mathbf{S_k},\mathcal{G})\cap\{A\}} + \varepsilon_{\mathbf{S_k}} \end{aligned}$$

The fact that the display above holds for every $k = 1, \ldots, K$ implies that the joint interventional distribution $P(X_\mathbf{S}|\mathrm{do}(x_A))$ satisfies

$$X_\mathbf{S} = \Lambda^{T}_{\mathbf{S},\mathbf{S}} X_\mathbf{S} + \Lambda^{\mathsf{T}}_{A,\mathbf{S}} x_A + \varepsilon_\mathbf{S}.$$

It follows that $X_\mathbf{S} = (I - \Lambda_{\mathbf{S},\mathbf{S}})^{-\mathsf{T}}(\Lambda^{\mathsf{T}}_{A,\mathbf{S}} x_A + \varepsilon_\mathbf{S})$ and since $Y \in \mathbf{S}$, we have

$$\tau_{AY} = \frac{\partial}{\partial x_A}\,\mathbb{E}[X_Y \mid \mathrm{do}(x_A)] = \Lambda_{A,\mathbf{S}}\left[(I - \Lambda_{\mathbf{S},\mathbf{S}})^{-1}\right]_{\mathbf{S},Y}.$$

Efficiency theory

Let Σ_n be the sample covariance. Consider the class of estimators

$$\mathcal{T} = \Big\{ \hat{\tau}(\Sigma_n) : \mathbb{R}_{\text{PD}}^{|\mathbf{V}|\times|\mathbf{V}|} \to \mathbb{R}^{|\mathbf{A}|} : \hat{\tau}(\Sigma_n) \text{ is a differentiable and consistent estimator of } \tau_{AY} \Big\}.$$

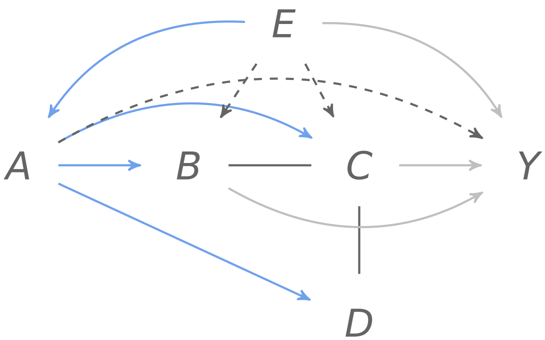
The efficiency theory entails two parts.

- Establish an efficiency bound on $\mathcal{T}$.
  The bound is derived from the gradient condition on $\mathcal{T}$ (as in standard semiparametric efficiency theory) and a **diffeomorphism**

  $$\mathbb{R}_{\text{PD}}^{|\mathbf{V}|\times|\mathbf{V}|} \longleftrightarrow ((\Lambda_{\text{pa}(\mathbf{B_k},\bar{\mathcal{G}}),\mathbf{B_k}}, \Omega_k) : k = 1, \ldots, K) \text{ associated with } \bar{\mathcal{G}},$$

  where $\bar{\mathcal{G}}$ is the saturated version of $\mathcal{G}$.
  This generalizes a result from Drton (2018).
- Verify that $\hat{\tau}_{AY}^{\mathcal{G}}$ achieves this bound.

Efficiency theory



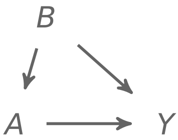
Saturated $\bar{\mathcal{G}}$ according to buckets.

$$\mathbf{B_1} = \{E\},\ \mathbf{B_2} = \{A\},\ \mathbf{B_3} = \{B, C, D\},\ \mathbf{B_4} = \{Y\}.$$

Proof sketch

1. Suppose $|\mathbf{A}| = 1$. Rewrite $\hat{\tau} \in \mathcal{T}$ as
$$\hat{\tau}(\Sigma_n) = \hat{\tau}\left((\hat{\Lambda}_k)_{k,\mathcal{G}}, (\hat{\Lambda}_k)_{k,\mathcal{G}^c}, (\hat{\Omega}_k)_k\right),$$
where $(\hat{\Lambda}_k)_{k,\mathcal{G}^c} = (\hat{\Lambda}_k)_{k,\bar{\mathcal{G}}\setminus\mathcal{G}}$ are introduced dashed edges.
2. Consistency of $\hat{\tau}$ implies
$$\frac{\partial \hat{\tau}}{\partial \hat{\Lambda}_{k,\mathcal{G}}} = \frac{\partial \tau_{\mathcal{G}}}{\partial \hat{\Lambda}_{k,\mathcal{G}}} \ (k = 2, \ldots, K), \quad \frac{\partial \hat{\tau}}{\partial \hat{\Omega}_k} = \mathbf{0} \ (k = 1, \ldots, K),$$
but $\frac{\partial \hat{\tau}}{\partial \hat{\Lambda}_{k,\mathcal{G}^c}}$ is free to vary.
3. Compute acov of $\left((\hat{\Lambda}_{k,\mathcal{G}})_k, (\hat{\Lambda}_{k,\mathcal{G}^c})_k\right)$ via asymptotic linear expansions.
4. By the delta method, an upper bound can be derived from quadratic form
$$\begin{aligned}\operatorname{avar}(\hat{\tau}) &= \begin{pmatrix} \frac{\partial \hat{\tau}}{\partial (\hat{\Lambda}_{k,\mathcal{G}})_k} \\ \frac{\partial \hat{\tau}}{\partial (\hat{\Lambda}_{k,\mathcal{G}^c})_k} \end{pmatrix}^{\mathsf{T}} \operatorname{acov}\left((\hat{\Lambda}_{k,\mathcal{G}})_k, (\hat{\Lambda}_{k,\mathcal{G}^c})_k\right) \begin{pmatrix} \frac{\partial \hat{\tau}}{\partial (\hat{\Lambda}_{k,\mathcal{G}})_k} \\ \frac{\partial \hat{\tau}}{\partial (\hat{\Lambda}_{k,\mathcal{G}^c})_k} \end{pmatrix} \\ &\leq \sup_{\partial\hat{\tau}/\partial(\hat{\Lambda}_{k,\mathcal{G}^c})_k} \begin{pmatrix} \frac{\partial \hat{\tau}}{\partial (\hat{\Lambda}_{k,\mathcal{G}})_k} \\ \frac{\partial \hat{\tau}}{\partial (\hat{\Lambda}_{k,\mathcal{G}^c})_k} \end{pmatrix}^{\mathsf{T}} \operatorname{acov}\left((\hat{\Lambda}_{k,\mathcal{G}})_k, (\hat{\Lambda}_{k,\mathcal{G}^c})_k\right) \begin{pmatrix} \frac{\partial \hat{\tau}}{\partial (\hat{\Lambda}_{k,\mathcal{G}})_k} \\ \frac{\partial \hat{\tau}}{\partial (\hat{\Lambda}_{k,\mathcal{G}^c})_k} \end{pmatrix}.\end{aligned}$$

What if we don't know the DAG?



DAG

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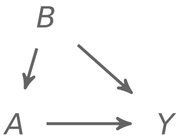

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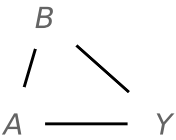

Completed Partially Directed Acyclic Graph (CPDAG)

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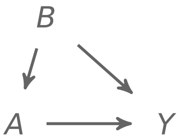

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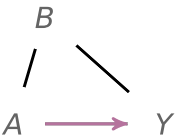

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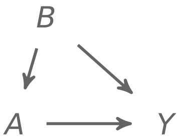

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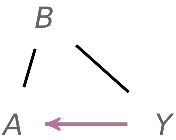

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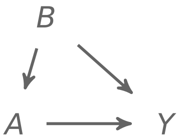

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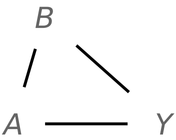

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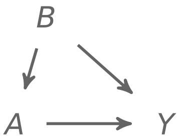

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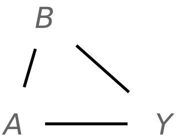

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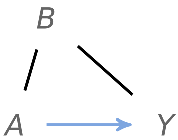
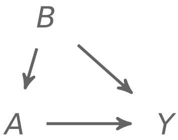
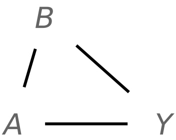


Maximally Oriented PDAG (MPDAG)

What if we don’t know the DAG?



DAG



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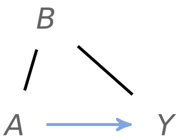
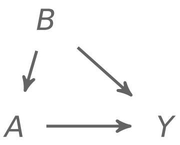


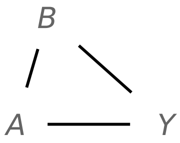
Maximally Oriented PDAG (MPDAG)

- A causal effect is **not always** identifiable from obs. data and a causal MPDAG.

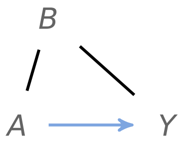
What if we don't know the DAG?



DAG



Completed Partially Directed Acyclic Graph (CPDAG)



Maximally Oriented PDAG (MPDAG)

- A causal effect is **not always** identifiable from obs. data and a causal MPDAG.

Graphical criterion	DAG	CPDAG	MPDAG
Adjustment (Pearl '93, Shpitser et al '10)	$\Rightarrow$		
Generalized Adjustment (Perković et al '15, '17, '18)	$\Rightarrow$	$\Rightarrow$	$\Rightarrow$
G-formula, Truncated Factorization (Robins '86, Pearl '93)	$\Leftrightarrow$		
Generalized G-formula (Perković '20)	$\Leftrightarrow$	$\Leftrightarrow$	$\Leftrightarrow$

$\Rightarrow$ - sufficient for identification,
$\Leftrightarrow$ - necessary and sufficient for identification